ground water

Issue Paper/

Using Hydraulic Head Measurements in Variable-Density Ground Water Flow Analyses

by Vincent Post[1], Henk Kooi[2], and Craig Simmons[3]

Abstract

The use of hydraulic head measurements in ground water of variable density is considerably more complicated than for the case of constant-density ground water. A theoretical framework for dealing with these complications does exist in the current literature but suffers from a lack of awareness among many hydrogeologists. When corrections for density variations are ignored or not properly taken into account, misinterpretation of both ground water flow direction and magnitude may result. This paper summarizes the existing theoretical framework and provides practical guidelines for the interpretation of head measurements in variable-density ground water systems. It will be argued that, provided that the proper corrections are taken into account, fresh water heads can be used to analyze both horizontal and vertical flow components. To avoid potential confusion, it is recommended that the use of the so-called environmental water head, which was initially introduced to facilitate the analysis of vertical ground water flow, be abandoned in favor of properly computed fresh water head analyses. The presented methodology provides a framework for determining quantitatively when variable-density effects on ground water flow need to be taken into account or can be justifiably neglected. Therefore, we recommend that it should become part of all hydrogeologic analyses in which density effects are suspected to play a role.

Introduction

Using hydraulic head observations to infer ground water flow directions and flow rates is a basic skill of every hydrogeologist. It is an application of Darcy's law: all that is required are estimates of hydraulic conductivity, K, and of the hydraulic gradient, ∇h, or components thereof. The practicality and convenience of the aforementioned field method obviously stems from the simple nature of Darcy's law. There are uncertainties in flow estimates, which arise from insufficient knowledge of hydraulic conductivity and heterogeneity, complications due to anisotropy, or large well spacing or screen length, but otherwise, it is rather straightforward.

What is less well known is the fact that the classical form of Darcy's law, cast in terms of hydraulic head, and hence, the intuitive field method, does not apply to ground water of variable density. Density variations can result from differences in temperature or pressure but more often are caused by differences in solute concentration. Variable density is particularly relevant in coastal areas, in sedimentary basins, and where dense contaminant plumes are present. The theory of ground water flow in variable density systems is considerably more complicated than under density invariant conditions, but there are still practical methods for dealing with it in combination with field data.

No simple guidelines currently exist that allow hydrogeologists to easily and robustly determine a priori whether a hydrogeologic analysis should be treated in a density-dependent or density-independent manner. The

[1]Corresponding author: Department of Hydrology and Geo-Environmental Science, Faculty of Earth and Life Sciences, Vrije Universiteit, De Boelelaan 1085, 1081 HV Amsterdam, The Netherlands; +31 (0) 20 598 7402; fax +31 (0) 20 598 9940; vincent.post@falw.vu.nl

[2]Department of Hydrology and Geo-Environmental Science, Faculty of Earth and Life Sciences, Vrije Universiteit, Amsterdam, The Netherlands.

[3]School of Chemistry, Physics and Earth Sciences, Flinders University, Adelaide, South Australia, Australia.

Received October 2006, accepted March 2007.

doi: 10.1111/j.1745-6584.2007.00339.x

analyses presented in this paper should be conducted where this uncertainty arises and in order to examine the potential consequences of ignoring density-dependent flow in hydrogeologic analyses.

Despite its great importance for hydrogeologists, there are few, if any, textbooks that explain how to correctly use field data to assess ground water flow under variable-density conditions. A theoretical treatment was presented by Lusczynski (1961), and although it was reiterated in some other publications (DeWiest 1967; Bear 1972; Oberlander 1989; St. Germain 2001), the proposed methodology did not become part of the mainstream literature. Excellent treatises, including examples, were presented by Van der Eem (1992) and Juster (1995), but these appear not to have reached a wide international audience.

Van Dam (1977) noted that "… there is much unacquaintedness and misunderstanding about the theory to be applied …" and Custodio (1987) stated that the various concepts of water heads of variable density "… do not solve the problem in a clear way." In a recent article, Simmons (2005) cautioned about the potential abuse of the fresh water head, noting, "Possibly one of the simplest analysis approaches used in variable density flow is the concept of 'equivalent freshwater head' but this is often too simple or even erroneous, especially where vertical flow is of interest." According to experience of the authors, confusion and misconceptions about the proper ways of dealing with density variations in flow calculations still abound, which forms the prime motivation for the present paper. The most common misconception observed by the authors (even in research papers and textbooks) is the notion that converting measured heads to fresh water heads suffices to analyze flow patterns and rates in variable-density ground water systems. This erroneous approach is not only caused by a lack of attention paid to variable-density flow calculations in textbooks but is probably also linked to the fact that several mainstream numerical codes that simulate variable-density flow solve ground water equations written in terms of fresh water head.

The theoretical framework as outlined in this paper is based on the existing literature. The objective here is to increase awareness among professionals in the field and thus avoid misinterpretation of head data in variable-density settings. The paper reinforces the appropriate methodology that should be applied in variable-density ground water analyses and discusses common problems that may be encountered along the way. Moreover, it will be argued that the use of the so-called environmental water head, as initially introduced by Lusczynski (1961), is best avoided. We develop "four golden rules" that we believe will provide useful and much needed guidance in the proper application of these concepts.

Emphasis is placed on situations where density is influenced by solute concentration since they are encountered by the vast majority of hydrogeologists. However, examples and procedures are readily transferable to conditions where temperature or pressure is the prime control on density variability by using the appropriate density contrasts. Since the aim here is to present the simplest set of practical rules for dealing with head measurements in variable-density systems, we intentionally refrain from presenting an analysis of the more complicated effects of anisotropy, heterogeneity, and dipping aquifers. Readers are referred to the works of Bachu (1995) and Bachu and Michael (2002) for a detailed discussion of these topics.

Fundamentals of Variable-Density Flow

Darcy's Law

The well-known short-hand notation of the differential equation form of Darcy's law is as follows:

$$\vec{q} = -K\nabla h \tag{1}$$

In terms of physics, this equation relates three quantities. $\vec{q}$ denotes specific discharge (volume of fluid per unit cross-sectional area of porous medium per unit time, $m^3/m^2/s$), also referred to as the Darcy velocity. ∇h is the driving force of ground water flow per unit weight of ground water (dimensionless). K is the hydraulic conductivity, a proportionality coefficient that describes the ease by which fluid flows through a porous medium per unit flow rate (m/s). As stated, we neglect directional dependency or anisotropy of the latter quantity in this manuscript and, therefore, treat K as a scalar quantity, which can be expressed by a singular numerical value at each point in the porous medium. Under these conditions, the three flow components are the following:

$$q_x = -K\frac{\partial h}{\partial x} \tag{1a}$$

$$q_y = -K\frac{\partial h}{\partial y} \tag{1b}$$

$$q_z = -K\frac{\partial h}{\partial z} \tag{1c}$$

Equation 1 is a simplified form of the more general physical law for fluid flow in a porous medium, which also applies to variable-density fluids (Bear 1972):

$$\vec{q} = -\frac{k}{\mu}(\nabla P - \rho\vec{g}) \tag{2}$$

with components:

$$q_x = -\frac{k}{\mu}\frac{\partial P}{\partial x} \tag{2a}$$

$$q_y = -\frac{k}{\mu}\frac{\partial P}{\partial y} \tag{2b}$$

$$q_z = -\frac{k}{\mu}\left(\frac{\partial P}{\partial z} + \rho g\right) \tag{2c}$$

where k is intrinsic permeability (m^2), a property of the porous medium; μ is dynamic viscosity (kg/m/s) of the ground water; P is fluid pressure ($kg/m/s^2$); ρ (kg/m^3) is fluid density; and $\vec{g}$ is the gravitational acceleration (m/s^2). Equation 2 explicitly shows the two basic driving forces for ground water flow: $-\nabla P$ is the force per unit volume of ground water due to spatial differences in pore water pressure, and $\rho\vec{g}$ is the gravity force per unit volume experienced by the ground water. Both forces, naturally, also act on ground water of uniform density. However, in Equation 1, these two forces are lumped into the single, convenient expression of gradient of hydraulic head, where the individual driving forces are rendered invisible. For ground water of variable density such a "gradient form" does not exist. This fundamental distinction is the main reason why quantification of ground water flow from field data, which normally occurs in the form of head measurements, requires a special treatment.

Head and Pressure Formulation

Equation 2 shows that for variable-density flow calculations, ground water pressure P and density ρ should be known, rather than hydraulic head h. Pressure, however, is not often used in everyday life, and hydrogeologists are more familiar with the concept of head. A number of key relationships among these quantities are therefore summarized here, which will subsequently be exploited to cast Equations 2a through 2c in terms of head.

Hydraulic head h (m) is obtained by measuring the level of the water-air interface in a ground water observation well, where levels refer to a common datum, often mean sea level. Two contributions to h are distinguished, and indicated in Figure 1a:

$$h_i = z_i + h_{p,i} \tag{3}$$

where z_i (elevation head) represents the (mean) level of the well screen, and $h_{p,i}$ (pressure head) is the length of the water column in the well relative to z_i. The subscript i is added to indicate that these values are measured at point i. For stagnant water conditions in the well, $h_{p,i}$ is related to the pressure of the ground water at the well screen P_i by the following:

$$h_{p,i} = \frac{P_i}{\rho_i g} \tag{4}$$

where ρ_i (kg/m^3) is the density of the water in the piezometer tube, i.e., of the ground water surrounding the well screen. It follows that, in a system where ρ varies spatially, values of $h_{p,i}$ do not correctly represent spatial variations of P. In other words, the same pressure can correspond to different values of $h_{p,i}$, depending on ground water density. Lusczynski (1961), therefore, used the term point water head for h_i to indicate that the values are uniquely linked to the ambient ground water density at the well screen.

To eliminate the ambiguity between $h_{p,i}$ and P_i, $h_{p,i}$ can be normalized using a reference density. That is, the water column in each observation well is replaced by an (imaginary) equivalent column of water of equal density for all the wells (Figure 1b). Any value of ρ can be used for this purpose (Van der Eem 1992), but fresh water is used most often, which gives rise to the definition of fresh water head:

$$h_{\mathrm{f},i} = z_i + \frac{P_i}{\rho_\mathrm{f} g} \tag{5}$$

where ρ_f is fresh water density. Fresh water head can be readily calculated from point water head measurements using the following:

$$h_{\mathrm{f},i} = \frac{\rho_i}{\rho_\mathrm{f}} h_i - \frac{\rho_i - \rho_\mathrm{f}}{\rho_\mathrm{f}} z_i \tag{6}$$

Fresh water head is always larger than or equal to point water head.

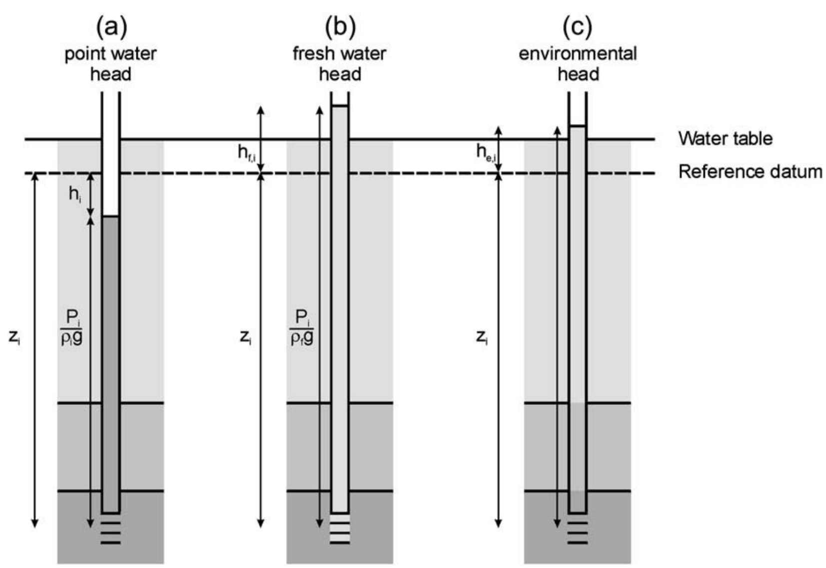

Figure 1. Schematic representation of head definitions in variable-density ground water systems (modified from Lusczynski 1961). Lightest shading corresponds to fresh water and darker shading represents increasing salinity.

From Equations 2a and 2b follows that horizontal flow components (q_x and q_y) should be calculated from the corresponding horizontal components of the pressure gradient. Alternatively, the horizontal component of the head gradient can be used, provided the heads refer to the same density. Rearranging and differentiating Equation 5 with respect to x and y and inserting the result into Equations 2a and 2b gives the following:

$$q_x = -\frac{k\rho_f g}{\mu_f}\frac{\mu_f}{\mu}\frac{\partial h_f}{\partial x} = -K_f\frac{\partial h_f}{\partial x} \tag{7a}$$

$$q_y = -\frac{k\rho_f g}{\mu_f}\frac{\mu_f}{\mu}\frac{\partial h_f}{\partial y} = -K_f\frac{\partial h_f}{\partial y} \tag{7b}$$

K_f is the fresh water hydraulic conductivity. It is assumed here that salinity variations have a negligible effect on μ so that $\mu_f/\mu \approx 1$ in Equations 7a and 7b, which is a very good approximation for most practical applications. Moreover, the difference between K_f and field-measured values of hydraulic conductivity, which are for ambient values of μ and ρ, is much smaller than the uncertainty associated with this parameter. Hence, no special corrections to existing hydraulic conductivity information are normally required.

Equation 2c shows that evaluation of the vertical flow component is different from the horizontal components in that a term involving local ground water density is needed. Similar to the horizontal flow components, the vertical component can also be cast in terms of fresh water head by rearranging and differentiating Equation 5 and inserting the result into Equation 2c:

$$\begin{aligned} q_z &= -\frac{k\rho_f g}{\mu_f}\frac{\mu_f}{\mu}\left[\frac{\partial h_f}{\partial z} + \left(\frac{\rho - \rho_f}{\rho_f}\right)\right] \\ &= -K_f\left[\frac{\partial h_f}{\partial z} + \left(\frac{\rho - \rho_f}{\rho_f}\right)\right] \end{aligned} \tag{7c}$$

in which the term $\frac{\rho - \rho_f}{\rho_f}$, which represents the relative density contrast, accounts for the buoyancy effect on the vertical flow. Equation 7c is used in several well-known variable-density flow and transport codes (e.g., MOCDENSE, SEAWAT).

Lusczynski (1961) introduced the concept of environmental water head ($h_{e,i}$) in order to calculate vertical flow with the convenient and familiar classical form of Darcy's law:

$$q_z = -\frac{k\rho_f g}{\mu_f}\frac{\mu_f}{\mu}\frac{\partial h_{e,i}}{\partial z} = -K_f\frac{\partial h_{e,i}}{\partial z} \tag{8}$$

The buoyancy effect on the vertical flow is taken into account in the definition of the environmental water head. In an appendix to his paper, Lusczynski (1961) demonstrated the validity of this approach. Figure 1c illustrates that environmental water head is obtained when the observation well is filled with stagnant water in which the variations of density are identical to those encountered along the vertical in the ground water just outside the well. That is, instead of point water or fresh water, the well is thought to be filled with "environmental" water. With this assumption and in the absence of vertical ground water flow, the water level in the well will coincide with the water table since the water pressure is hydrostatic both inside and outside the well. If there is vertical flow, the water pressure outside the well will differ from the hydrostatic pressure. In Lusczynski's (1961) definition of the environmental water head, this difference is expressed as a column of fresh water, which is a measure for the deviation of $h_{e,i}$ from the water table. The concept is ingenious but unfortunately becomes nonintuitive when high-density water is present all the way up to the water table.

DeWiest (1967) introduced the "true environmental head" in which environmental water head is related to pressure according to the following:

$$h_{e,i} = z_i + \frac{p_i}{\rho_e g} \tag{9}$$

where ρ_e is the average density of the water between z_i and $h_{e,i}$ *inside* the well. This definition is not very practical, however, since $h_{e,i}$ and ρ_e are interdependent (Juster 1995). Moreover, ρ_e is easily confused with the average density of the water *outside* the well ρ_a (defined later on in this paper) used by Lusczynski (1961) in his original definition of environmental water head.

Application and Interpretation Procedure

In this section, the procedure for the interpretation of head measurements in variable-density ground water will be outlined. These will be illustrated with examples and the implications of the necessary assumptions will be discussed.

Horizontal Flow Component

When calculating horizontal flow, it is crucially important that the fresh water head gradient in Equations 7a and 7b (or pressure gradient in Equations 2a and 2b) is evaluated using fresh water heads at the same depth because, in contrast to uniform density ground water, fresh water head may vary with depth, even for hydrostatic (i.e., no vertical flow) conditions. Thus, when measurements are taken from piezometers with screens at different depths, fresh water heads need to be calculated at a suitable reference depth. A common approach is to assume hydrostatic conditions between the well screen and the reference depth. The pressure at the reference depth (z_r) then becomes as follows:

$$P_r = P_i - g\int_{z_i}^{z_r}\rho \mathrm{d}z = P_i - \rho_a g(z_r - z_i) \tag{10}$$

with

$$\rho_a = \frac{1}{z_r - z_i}\int_{z_i}^{z_r}\rho \mathrm{d}z \tag{11}$$

ρ_a denotes the average water density between measurement point z_i and the reference level z_r. The corresponding fresh water head at z_r ($h_{f,r}$) is then obtained from Equation 5:

$$h_{f,r} = z_r + \frac{P_r}{\rho_f g} = z_r + \frac{\rho_i}{\rho_f}(h_i - z_i) - \frac{\rho_a}{\rho_f}(z_r - z_i) \quad (12)$$

Using field data, the horizontal component of flow is obtained from the following:

$$q_x = -K_f \frac{\Delta h_f}{\Delta x} \quad (13)$$

and analogously for q_y.

Example 1. Horizontal Flow

Consider two piezometers some distance Δx apart that have their well screens located in the same aquifer (Figure 2). Screen depths and the measured point water heads and densities are listed in Table 1. Because the screen depths differ 10 m, fresh water heads have to be calculated for a single reference depth using Equation 12. Results for $z_r = -40$ m (the depth of piezometer 1) and $\rho_a = 1005$ kg/m^3 (average of the densities at the two screens) are given in Table 2 under the heading "mean." Comparison of the values with the point water heads in Table 1 shows that the horizontal gradients and hence the suggested flow directions for the two head types are opposite. Clearly, the gradient of $h_{f,r}$ should be combined with a value for K_f to arrive at a proper estimate of horizontal flow.

It is important to realize that several of the steps outlined in example 1 involve assumptions and that each of these assumptions introduces uncertainty in the final flow estimate. A potential source of uncertainty specific for variable-density flow estimation is the required estimate of average density ρ_a between screen and reference depth. A simple approach to (approximately) quantify this uncertainty can be demonstrated for the data of example 1. Above, the average of the densities of the water at the two screens was used for ρ_a, in order to obtain a fresh water head value for the 50-m-deep well at $z_r = -40$ m. This average density can be thought to correspond to a linear vertical density profile in the depth range between the two screens as shown in Figure 2. Evidently, other density profiles and corresponding values of ρ_a are possible. In the absence of additional constraints, two "end-member" density profiles can be constructed, indicated by "min" and "max" in Figure 2, where values of ρ_a correspond to the density values listed for both piezometers (Figure 2). Minimum, maximum, and mean values for $h_{f,r}$ are listed in Table 2. Results show that the head difference between the two piezometers varies with the assumed average density by about 40%, which implies a similar uncertainty in the magnitude of the flow component. Strictly speaking, the uncertainty may still have been underestimated with the adopted approach because the density at -40-m depth at piezometer 2 need not be equal to the density at piezometer 1 if lateral variations in density between the two wells occur. Therefore, alternative methods of uncertainty assessment are possible. The present example does demonstrate, however, the importance of conducting such an assessment to check to what extent inferred flow conditions are significant.

Vertical Flow Component

The second term in large brackets in Equation 7c is essential to correctly describe variable-density flow. For example, under hydrostatic conditions ($q_z = 0$) in a saline aquifer of sea water concentration ($\rho = 1025$ kg/m^3), the density excess ratio is $(\rho - \rho_f)/\rho_f = 0.025$, and fresh

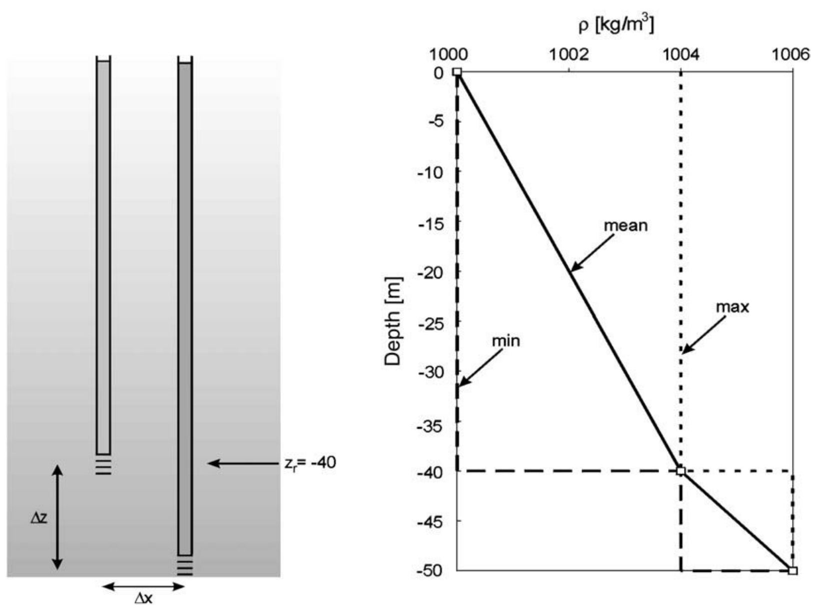

Figure 2. Left: Piezometers used in example problems. Darker shading represents increasing salinity. Note that in the examples, only the density at the well screens is known and not the true density distribution in the aquifer. Right: vertical density distributions considered in the calculations of $h_{f,r}$ and $h_{e,i}$.

Table 1
Well Screen Depth, Point Water Head, Density, and Calculated Fresh Water Heads of Example 1

Piezometer	Screen Depth (m)	h_i (m)	ρ (kg/m^3)	$h_{f,i}$ (m)
1	−40	1.25	1004	1.42
2	−50	1.20	1006	1.51

water head decreases with depth according to $\partial h_f/\partial z = -0.025$. Thus, both terms cancel each other in Equation 13 to correctly describe the zero flow condition. Ignoring the buoyancy term, however, would yield a markedly erroneous flow estimate.

Example 2. Vertical Flow: Fresh Water Head Formulation

For the calculation of vertical flow, consider the same piezometers as in example 1. In this example, however, their well screens are not separated laterally but are in the same vertical ($\Delta x = 0$). To evaluate the vertical flow component, Equation 7c must be cast in finite-difference form:

$$q_z = -K_f\left[\frac{\Delta h_f}{\Delta z} + \left(\frac{\rho_a - \rho_f}{\rho_f}\right)\right] \tag{14}$$

where $\Delta h_f = h_{f,2} - h_{f,1}$ and $\Delta z = z_2 - z_1$ are the difference in fresh water head and elevation head of the piezometers, respectively, and ρ_a is the average density of the ground water between the screens, defined analogously to Equation 11. The calculated values of q_z for a value of $K_f = 10$ m/d are listed in Table 3. As before, uncertainty arises from the unknown average density between the point measurements (Figure 2).

For coastal settings where fresh overlies saline ground water, Lusczynski (1961) inferred the following:

$$h_{e,i} = z_r + \frac{\rho_i}{\rho_f}(h_i - z_i) - \frac{\rho_a}{\rho_f}(z_r - z_i) \tag{15}$$

where z_r denotes an arbitrary reference level above which ground water is fresh and where ρ_a is the average density of water between z_r and screen depth z_i. The latter is calculated with Equation 11.

Table 2
Fresh Water Heads of Piezometers at Reference Depth $z_r = -40$ m of Example 1 for $\rho_a = 1004$ (Minimum), $\rho_a = 1005$ (Mean), and $\rho_a = 1006$ kg/m^3 (Maximum)

		$h_{f,r}$		
Piezometer	z_r (m)	Minimum (m)	Mean (m)	Maximum (m)
1	−40	1.42	1.42	1.42
2	−40	1.47	1.46	1.45

Table 3
Fresh Water Head Gradient, Buoyancy Term, and Vertical Component of Specific Discharge of Example 2 for $\rho_a = 1004$ (Minimum), $\rho_a = 1005$ (Mean), and $\rho_a = 1006$ kg/m^3 (Maximum)

	Minimum	Mean	Maximum
$\frac{\Delta h_f}{\Delta z}$	9×10^{-3}	9×10^{-3}	9×10^{-3}
$\frac{\rho_a - \rho_f}{\rho_f}$	0.004	0.005	0.006
q_z (m/d)	0.05	0.04	0.03

Example 3. Vertical Flow: Environmental Water Head Formulation

Again, consider the same piezometers as in example 2. Application of Equation 15 assuming $z_r = 0$ m (to make sure reference depth is above the domain of "non-fresh water" for minimum, mean, and average density distributions) results in the values listed in Table 4. Values of q_z are calculated with the finite-difference form of Equation 8:

$$q_z = -K_f\frac{\Delta h_{e,i}}{\Delta z} \tag{16}$$

Calculated values of q_z are the same as in example 2, as they should be.

The expression for $h_{e,i}$ in Equation 15 is identical to that of $h_{f,r}$ in Equation 12. The values of $h_{f,r}$ listed in Table 2 indeed yield the vertical gradients in environmental water head listed in Table 4. Values of $h_{e,i}$ and $h_{f,r}$ do differ, however, because different reference levels were used (0 and −40 m, respectively). A constant difference in head is of no consequence, however, when gradients are calculated.

The equivalence of the environmental water head approach and the fresh water head approach to calculate vertical flow can furthermore be demonstrated in the following way. Equation 15 is written in terms of fresh water head using Equations 5 and 10:

$$h_{e,i} = h_{f,r} = h_{f,i} - \frac{\rho_a - \rho_f}{\rho_f}(z_r - z_i) \tag{17}$$

At two well screens in the same vertical, this gives the following:

$$h_{e,1} = h_{f,1} - \frac{\rho_{a,1} - \rho_f}{\rho_f}(z_r - z_1) \tag{18a}$$

$$h_{e,2} = h_{f,2} - \frac{\rho_{a,2} - \rho_f}{\rho_f}(z_r - z_2) \tag{18b}$$

Note that the average densities are different for each screen because they are evaluated over different intervals and that:

$$\rho_{a,1}(z_r - z_1) = \int_{z_1}^{z_r} \rho \mathrm{d}z \tag{19a}$$

$$\rho_{a,2}(z_r - z_2) = \int_{z_2}^{z_r} \rho \mathrm{d}z = \rho_{a,1}(z_r - z_1) + \rho_a(z_1 - z_2) \tag{19b}$$

where ρ_a is the average density between the two screen depths. Taking the difference ($\Delta h_{e,i} = h_{e,2} - h_{e,1}$) yields the following:

$$\begin{aligned}\Delta h_{e,i} &= h_{f,2} - h_{f,1} + \frac{\rho_a - \rho_f}{\rho_f}(z_2 - z_1) \\ &= \Delta h_{f,i} + \frac{\rho_a - \rho_f}{\rho_f}\Delta z\end{aligned} \tag{20}$$

Dividing by Δz gives the term in large brackets in Equation 14.

Discussion and Conclusions

The previous discussion demonstrates that very subtle density variations can have a major impact on the flow field and necessitate employment of the methodology outlined earlier. A detailed description of the full density field in the area under investigation is costly and difficult to obtain and will usually not be available. At minimum though, all head measurements should be carried out in conjunction with measurements of electrical conductivity. The density can then be estimated from simple relationships between density and salinity available in the literature (Reilly and Goodman 1985; Holzbecher 1998).

So far, it has not been assessed at what density contrasts the aforementioned corrections become significant and need to be taken into account. To this aim, the value of $h_{f,i}$ (Equation 6) for $h_i = 0$ is contoured in Figure 3 as a function of ρ_i and z_i. It provides a measure of the degree of misinterpretation of fluid pressure at the well screen when solely relying on point water heads for typical conditions in coastal aquifers. As can be seen from the figure,

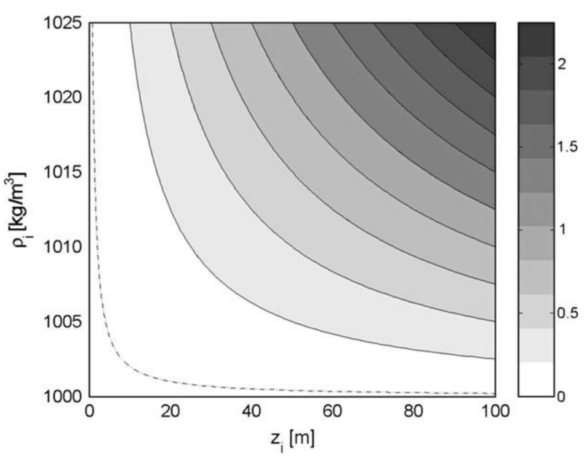

Figure 3. Contour plot of the value of $h_{f,i}$ (in m) according to Equation 6 for $h_i = 0$ m as a function of ρ_i and z_i. The dashed line represents the minimum error associated with head measurements (0.02 m).

Table 4
Environmental Water Heads of Piezometers at $z_i = -40$ and $z_i = -50$ m, Environmental Water Head Gradient and Vertical Component of Specific Discharge of Example 2 for Different Values of ρ_a

	Minimum	Mean	Maximum
$h_{e,-40}$	1.42	1.34	1.26
$h_{e,-50}$	1.47	1.38	1.29
$\frac{\Delta h_e}{\Delta z}$	5×10^{-3}	4×10^{-3}	3×10^{-3}
q_z (m/d)	0.05	0.04	0.03

h_f deviates up to several meters from the "measured" value of $h_i = 0$. Note that in flat coastal areas (e.g., deltas, sedimentary basins), head differences that drive ground water flow are typically on the order of decimeters.

The line in Figure 3 represents the precision of carefully taken head measurements (0.02 m) and can be used to assess when density variations start to become important. Slight deviations from fresh water densities may already lead to corrections exceeding the precision of the head measurements at depths of several tens of meters or more. Assessments of this type should always be the first step in any hydrogeologic study to determine if variable-density effects need to be taken into account. If these effects cannot be ruled out a priori, they should be quantified by means of the analyses presented in this paper. In order to justify the choice of a conceptual hydrogeologic framework in an explicit rather than implicit manner, such checks should be applied more routinely than they appear to be at present.

It was already discussed previously that the required estimate of the average density ρ_a between screen and reference depth is a significant source of uncertainty in the flow calculations. Uncertainties obviously tend to increase considerably for larger vertical distances between screen depth and reference level. As a general rule, reference depth should, therefore, be chosen within the depth range of the employed well screens. Contour maps of fresh water head in extensive aquifers, even when referred to the same vertical level, should therefore be regarded with suspicion and subsequently used with caution. The requisite of using a single horizontal reference level further implies that isohyps maps for tilted aquifers cannot be constructed over large distances in the dip direction.

Other potential sources of uncertainty in these analyses are (1) estimates of hydraulic conductivity, K_f, which is beyond the scope of the present paper because it is common also to uniform density ground water flow; (2) the assumption of hydrostatic conditions between screen and reference depth; and (3) finite-length screens. Additional complexity (and uncertainty) will occur in systems where there is substantial heterogeneity, anisotropy, and complex geometries associated with, for example, sloping aquifer configurations.

The third source of uncertainty noted previously, finite length of well screens, stems not only from ambiguity in assigning a single value depth to the screen but also from additional uncertainty regarding the meaning of the density of water obtained from the well due to uncertain mixing conditions along the well screen. Assessment of the impact of these uncertainties appears nontrivial but at least suggests that variable-density ground water flow assessment using data from wells with long screens should be avoided.

Although the environmental water head approach has the nicety of a simpler and more familiar expression of the flow Equation 16, there is no true advantage over Equation 14. Whereas in the latter approach density corrections are applied to the gradient component, in the environmental water head approach, similar corrections are incorporated in the calculation of environmental water heads before the gradient operator is applied. Environmental water heads may seem more practical because they are more readily rendered in the form of isohyps maps or vertical cross sections. However, such maps are hardly useful and may even be considered "dangerous" because they easily cause mis- or overinterpretation and do not allow visualization of the often large uncertainties. Furthermore, the choice of reference depth in settings where salt water overlies fresh water or where saline surface water is present, such as in estuarine and offshore ground water hydrology, is nonintuitive, not described in literature, and, therefore, less practical for nonspecialists. It can therefore easily be argued that the fresh water head approach with the appropriate correction for negative buoyancy should be endorsed as the preferred approach in variable-density analyses because it only requires making assumptions about the water density variations between well screens.

The procedures and guidelines set forward in this paper can be summarized as a set of four golden rules that should be adhered to in order to correctly infer ground water flow (directions and magnitudes) of variable-density ground water, namely:

1. Collect fluid density information with all head measurements.
2. Calculate horizontal flow components from fresh water heads referenced to the same elevation.
3. Calculate vertical flow components from the gradient of fresh water head with an appropriate correction for (negative) buoyancy of the ground water between the two measurement depths.
4. Provide an assessment of the uncertainty associated with the estimated ground water flow components.

Acknowledgment

The authors would like to thank the reviewers Christian Langevin and Ann Mulligan for their thoughtful and constructive comments, which helped improve the manuscript.

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